Fast Hotspot Analysis Applying Physics-based Electromigration Models

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***Abstract*—The problem of electromigration (EM) in integrated circuits has been addressed so far by imposing current-density limits. Yet, physics-based modeling allows more precise, and thus increased, current-density limits. However, this novel method is still not used in industrial design flows due to its multitude of requirements. To ease its long overdue application, we provide a simple metric by treating long wire segments as semi-infinite lines. This allows us to obtain the lifetime of individual nodes without extensive calculation. Exploiting the stress impact length, we derive a criterion to check whether wire segments are long enough to apply our metric. For the remaining segments not fulfilling this criterion, we propose the use of simplified RC equivalent circuits for local EM analysis. Both methods allow the application of precise physics-based EM models with low effort, hopefully easing their use in industrial EM verification.**

***Index Terms*—Electromigration, Stress, Verification, Current Density, Lifetime**

I. Introduction

Within the last years, electromigration (EM) verification of integrated circuit (IC) layouts experienced a paradigm shift from current-density limits toward physics-based stress modeling [1]. However, this paradigm shift has only affected the academic research community while current-density verification remains the industry standard.

Current-density verification is thus well-established and proven to be reliable in practice. It is straightforward for engineers to implement, and moreover, commercial tools are available for rapid full-chip current-density verification. Yet, research has shown that this approach is overly pessimistic, leading to unnecessarily strict design constraints [2].

Physics-based modeling, on the other hand, enables precise EM lifetime predictions. Based on hydrostatic-stress calculation [3], it indicates locations where the risk of EM degradation is high and, thus, countermeasures can be effectively installed.

In this work, we propose a new and easy-to-apply criterion for fast, yet physics-based EM-lifetime verification of interconnect nodes. In contrast to existing physics-based methods, it does not require complex calculations – such as solving partial differential equations – thus overcoming one of the major challenges associated with these methods. Its simplicity is comparable to the current-density checks that are widely used for EM verification. As visualized in Fig. 1, we introduce

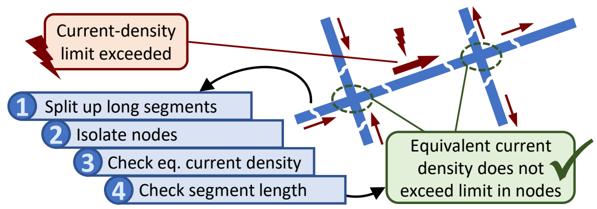

Fig. 1. Our concept of node-wise EM verification for segments fulfilling our length criterion. It allows simple, yet accurate EM verification and locally increased current-density limits.

splitting points in long wire segments that only experience stress growth at their ends, treating them as semi-infinite lines. Our method can allow higher current densities for individual segments compared to the global current-density limit provided in the process design kit (PDK) offering direct benefit to critical analog or power supply lines.

Our method is limited to segments of a minimum length which is checked by our novel stress length criterion. For shorter segments, we show, how the previously published method of equivalent RC circuits [4]–[6] for stress evolution simulation can benefit from our approach. We thus enable EM hotspot analysis with reduced RC model size.

In summary, the contributions of this paper are:

- A stress length criterion to check whether a segment is long enough to be split up (Sec. III-A).
- An EM check for interconnect nodes fulfilling the stress length criterion with a simplicity comparable to a standard current-density limit (Secs. III-B and III-C).
- Optimized RC equivalent circuits for transient EM simulation of interconnects with short segments to reduce the model size (Sec. III-D).

II. State of the Art

A. Electromigration Models

First, we introduce the industry practice of current-density verification as a foundation for understanding physics-based modeling. There are two types of current-density limits:

- general, Black-based current-density limits [7] and
- short-length limits based on the Blech criterion [8].

The well-known *Black equation* calculates the lifetime t_{life} of a single-segment wire depending on the current density j, temperature T and the activation energy E_{a} with sufficiently

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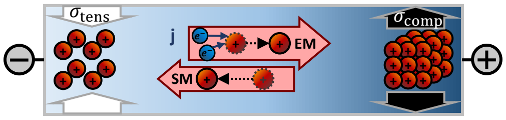

Fig. 2. EM and SM in a finite line causing stress evolution.

accurate results. However, applying it to multi-segment wires is highly pessimistic as we will discuss in Sec. IV.

The so-called Blech product $(jL)_{\mathrm{Blech}}$ with L being the wire length ensures immortality. Based on this, PDKs allow higher current densities for short wires. The concept can also be extended to multi-segment interconnects as proposed in [9].

In research, on the other hand, the focus is on physics-based migration models that characterize the diffusion processes occurring within an interconnect and the consequent evolution of hydrostatic stress (Fig. 2). In this framework, the traditional criterion of maximum current density is replaced by a critical stress threshold for failure, denoted as σ_{crit}. The lifetime of an interconnect is defined as the time when σ_{crit} is reached. Physics-based lifetime verification is based on the so-called Korhonen equation [3] and its extensions to multi-segment interconnects. This equation describes the evolution of stress, σ, for a finite single-segment line of length, L, over time, t, with boundary conditions (BCs) as follows:

$$\frac{\partial\sigma}{\partial t} = \frac{\partial}{\partial x}\left[\kappa\left(\frac{\partial\sigma}{\partial x} - \beta j\right)\right], \text{ BCs: } \left.\frac{\partial\sigma}{\partial x}\right|_{x=0,L} = \beta j \quad (1)$$

$$\text{where } \kappa = DB\Omega/k_{\mathrm{B}}T \text{ and } \beta = e\rho Z/\Omega \quad (2)$$

with diffusivity $D = D_0 \cdot \exp\left(-E_{\mathrm{a}}/\left(k_{\mathrm{B}}T\right)\right)$; here, D_0 is the diffusion constant, B the Bulk modulus, Ω the atomic volume, k_{B} the Boltzmann constant, e the elementary charge and Z the effective charge number.

B. Stress Evolution in Semi-infinite Lines

Regarding EM, nets are typically verified within a single metal layer, since the diffusion barriers located beneath the vias effectively block atomic flux between layers. Consequently, stress evolution in interconnect segments separated by these barriers occurs independently.

The work in [3] provides solutions for calculating stress evolution for two special cases: finite and semi-infinite lines. Their main difference lies in the back stress: In a finite line, stress migration (SM) will eventually cause a flux equilibrium (steady state) where the stress does not grow further. This is not the case for semi-infinite lines. For a semi-infinite line with the diffusion barrier at $x = 0$, [3] derives the stress as

$$\sigma(x,t) = \beta j\left[\sqrt{\frac{4\kappa t}{\pi}} \cdot \exp\left(\frac{-x^2}{4\kappa t}\right) - x \cdot \operatorname{erfc}\left(\frac{x}{\sqrt{4\kappa t}}\right)\right] \quad (3)$$

Here, the stress will only build up at the start of the line. For $x \to \infty$, the stress will be zero.

III. Method of Hotspot Analysis

Our subsequently presented approach enables quick EM lifetime checks for individual nodes or sections of a net. While most physics-based EM simulation methods consider the whole metal structure within a metal layer (with diffusion barriers as splitting points of the net), we introduce additional splitting points in long wire segments as shown in Fig. 3.

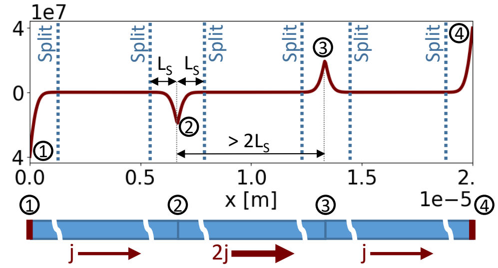

Fig. 3. Illustration of our concept to split long wire segments at the stress impact length L_{S} where the stress remains zero (Sec. III-A). Thus, the EM robustness of all nodes can be assessed individually, treating the split segments as semi-infinite lines (Secs. III-B and III-C).

We define a *node* as a point of an interconnect where the current density and/or the wire width changes. An *interconnect*, in this context, is a metal structure within one metal layer consisting of one or more segments. A *segment* is a wire section between two adjacent nodes. In a Manhattan layout, a node can connect one up to four segments of an interconnect.

If a wire segment is long enough, the stress at its ends behaves like in two independent semi-infinite lines within the wire's lifetime. This is experimentally supported by the well-known fact that in long wires t_{life} is independent from the wire length [10]. The reason for this is that stress starts to grow at the segment's ends and only gradually evolves toward the middle of the segment. Thus, if stress evolution does not start in the middle of the wire segment before the critical stress is reached in the net, the segment can be split into two parts that will both be treated as semi-infinite lines.

A. Estimating the Stress Impact Length

In order to decide whether a segment can be treated as a semi-infinite line, we have to consider the portion of the wire that is impacted by stress evolution at $t = t_{\mathrm{life}}$. We call this the *stress impact length* L_{s}. This is closely related to the well-known *diffusion length* $L_{\mathrm{diff}} = \sqrt{\kappa t}$ [3] which describes the average distance that atoms move. We propose a conservative criterion that allows the quantification of the minimum stress decrease $\alpha = \sigma(L_{\mathrm{s}}, t_{\mathrm{life}})/\sigma(0, t_{\mathrm{life}})$.

The equation for stress evolution in a semi-infinite line (3) consists of two parts, an exponential and a complementary error function where for $x \geq 0$ it can be shown that

$$\left|\sqrt{4\kappa t/\pi} \cdot \exp\left(-x^2/4\kappa t\right)\right| > \left|x \cdot \operatorname{erfc}\left(x/\sqrt{4\kappa t}\right)\right|. \quad (4)$$

Consequently, the left side of Eq. (4) yields a conservative stress result σ_{cons} which is close to the actual stress σ, but always slightly higher. We can calculate σ_{cons} as

$$\sigma_{\mathrm{cons}} = \beta j\left[\sqrt{4\kappa t/\pi} \cdot \exp\left(-x^2/4\kappa t\right)\right]. \quad (5)$$

This equation has the form of a scaled normal distribution with a mean value $\mu = 0$

$$f(x) = a/\sqrt{2\pi\gamma^2} \cdot \exp\left(-x^2/2\gamma^2\right) \quad (6)$$

where a is a scaling factor and γ is the standard deviation.

Comparing Eqs. (5) and (6), we find that $a = 4\beta j\kappa t$ and $\gamma = \sqrt{2\kappa t}$. Thus, we can exploit the known characteristics of the normal distribution and set L_{S} to a suitable multiple of γ. We suggest using the 3γ-point (where the PDF reduces to $\approx 1.1\%$ of its peak value, i. e., $\alpha = 1.1\%$) and will continue using it throughout this paper. Thus, we set

$$L_{\mathrm{S}} = 3\sqrt{2\kappa t}. \tag{7}$$

Please note that stress is growing at both ends of a wire segment. Consequently, as shown in Fig. 3, the criterion for treating a segment as two independent semi-infinite lines is

$$L_{\text{segment}} \geq 2 \cdot L_{\mathrm{S}} = 6\sqrt{2\kappa t}. \tag{8}$$

The applicability of the approach presented in this paper is highly dependent on L_{S} and its ratio to the typical segment length of the respective technology. The smaller L_{S} (and the higher the typical segment length), the more likely it is that the stress length criterion is fulfilled and fast lifetime estimation can be performed.

According to Eq. (7), the necessary L_{S} is dependent on κ (including T and E_{a}, cf. (2)) and t (the specified lifetime). There is no direct dependency on the technology node (like wire width).

Obviously, the stress impact length L_{S} grows with increasing t_{life} and T. The largest impact, however, has E_{a}. For a high E_{a}, L_{S} is low. This supports the application of our approach, as semiconductor fabs aim to maximize E_{a} in their technologies. The reason for this is that a high E_{a} allows significantly increased EM boundaries for the permissible current density.

B. Current-density Limit for a Long Wire

First, L_{S} is calculated by setting $t = t_{\text{life}}$. As the required t_{life} is the same for all nodes, L_{S} is a constant for the whole design.

A long wire (according to (8)) can be treated as a semi-infinite line. Thus, by setting $x = 0$ and $\sigma = \sigma_{\text{crit}}$ in Eq. (3), we can solve for j to obtain the maximum permissible current density $j_{\max}$ that fulfills lifetime requirements:

$$j_{\max} = \sqrt{\pi/(4\kappa t_{\text{life}})} \cdot \sigma_{\text{crit}}/\beta. \tag{9}$$

This is corresponding to the current-density limit obtained by the empirical Black equation.

Note that our approach does not reduce design pessimism for single-segment finite wires; however, it provides valuable insight into its origins. In the next section, we extend the method to multi-segment interconnects and demonstrate how it can enable higher permissible current densities without compromising lifetime constraints.

C. Current-density Limit for Nodes

Consider our example in Fig. 3 where the stress at nodes 2 and 3 is lower that at nodes 1 and 4 and, thus, clearly not determined by the high current density $2j$ in the connecting segment. This leads to the important (but not new) conclusion that an interconnect's t_{life} is not necessarily determined by the maximum current density, as assumed in the Black-based current-density verification. This has been experimentally

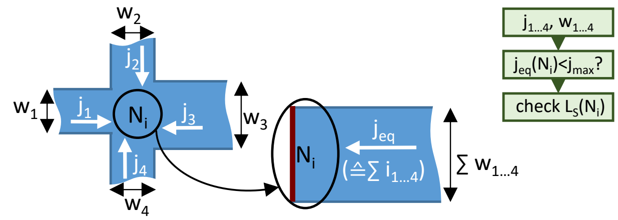

Fig. 4. A node (left) and the procedure of its EM assessment (right). The wide semi-infinite wire (middle) visualizes how to calculate j_{eq}.

demonstrated in [2], which also proposes the concept of an efficient node current density. However, revisiting this concept with the background of stress evolution, we propose a more precise approach to calculate the equivalent current density j_{eq} of a node. Also, we explored the stress impact length in the previous sections and can thus improve the original Via Node Vector Method [2] by a concise definition of when a segment is considered to be *long*.

For a node, j_{eq} is the sum of all currents flowing into it[1] divided by the total cross-section of all connected wire segments. The idea behind this is positioning all segments next to each other to connect them to one (wider) semi-infinite wire where the node is the common starting point (cf. Fig. 4, middle). By definition, we treat currents flowing into the node as positive and currents flowing out of the node as negative. The equivalent current density can be calculated as

$$j_{\text{eq}} = \frac{\sum_{i=0}^{n} i_i}{\sum_{i=0}^{n} hw_i} = \frac{\sum_{i=0}^{n} hw_i j_i}{\sum_{i=0}^{n} hw_i} = \frac{\sum_{i=0}^{n} w_i j_i}{\sum_{i=0}^{n} w_i} \tag{10}$$

with h being the wire (i. e. metal layer) thickness[2].

At this point, the node can be treated as a semi-infinite line, as described in Sec. III-B. It is important to note that the stress length criterion must be satisfied for all segments connected to the node. Under these conditions, stress will accumulate uniformly across the segments (cf. Fig. 3).

D. How to Handle Short Segments

If a segment is too short to fulfill the stress length criterion, the stress evolution of the two nodes connected by this segment is not independent. Thus, their EM robustness cannot be assessed individually using the equivalent current density.

However, our approach simplifies these lifetime calculations as well: While other methods calculate stress in the whole interconnect, our stress length criterion can help splitting it into smaller sections supporting localized EM checks for segments that exceed the Black-based current-density limit.

We will demonstrate this using the example of equivalent RC circuits [4]–[6]. We will not go into the details on how to build equivalent RC networks; the basic principle is shown in

[1]This is not to be confused with Kirchhoff's Current Law as there can be vias connected to this node that lead currents into other metal layers. These will not longer be considered for EM verification in the metal layer of the node as these currents will pass a diffusion barrier.

[2]Eq. (10) is consistent with prior EM solutions. For a junction of two segments, it corresponds to the "stress wave" from the node in [11], where the effective current of this stress wave is consistent with (10). For junctions with more than two wire segments, the RC circuit equivalent [4] implies that the current source at the node is similar to the numerator of (10). This current is symmetrically distributed to each incident wire, and, hence, the effective current in each wire is obtained by dividing the numerator by the denominator.

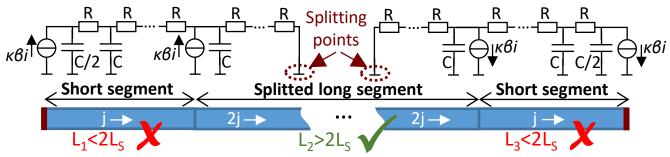

Fig. 5. Example of how to transform a 3-segment interconnect into two simplified equivalent RC circuits. The splitting points are realized by connecting the corresponding circuit nodes to ground.

Fig. 5. By construction, the voltage in this equivalent circuit equals the hydrostatic stress in the interconnect. Thus, applying transient SPICE simulation, EM stress can be simulated.

To simplify the network used to evaluate individual critical short segments we exploit that in long segments only the section within the stress impact length is relevant for stress evolution (while the rest of the interconnect remains at $\sigma = 0$). Thus, we can again split up the interconnect into smaller pieces that can consist of more than one node with its connected segments. We can find the split points on long segments fulfilling the stress length criterion. We will split these segments in a distance of L_S from their start node and connect the split points to ground ($V = 0$, corresponding to $\sigma = 0$) as illustrated in Fig. 5.

Hence, long segments do not have to be simulated in their entirety, we reduce the simulation to the area of interest (where stress will evolve). Additionally, we can now reduce the simulation of large multi-segment interconnects to the optimized RC networks representing hotspot locations where the current-density limit from the PDK is exceeded.

IV. Results and Discussion

We tested our node lifetime calculation by comparing FEM simulations of a node with the equivalent simulation of a semi-infinite line stressed with j_{eq}. To demonstrate this with two examples, we set up simulations of a node with two connected segments. The two segments both have diffusion barriers on their other end. We defined $E_a = 1\,\text{eV}$ and $T = 100\,°\text{C}$ resulting in $\kappa = 1.19 \cdot 10^{-20}\,\text{m}^2/\text{s}$. For a targeted lifetime of 5 years, this results in $L_S = 5.81\,\mu\text{m}$, so we defined the segment length of our models to $12\,\mu\text{m}$.

In the first example, we set $w_1 = w_2 = 50\,\text{nm}$, $h_1 = h_2 = 100\,\text{nm}$, and $i_1 = i_2 = 0.2\,\text{mA}$. This results in a current density of $j_1 = j_2 = 4\,\text{MA/cm}^2$ where both currents flow into the node. Applying Eq. (10), we can calculate $j_{eq} = 4\,\text{MA/cm}^2$. For both the node and the semi-infinite line stressed with j_{eq}, we obtain a lifetime $t_{life} = 3.68\,\text{years}$ which does not fulfill the lifetime criterion of 5 years.

To increase the node's lifetime, in the second example, we vary the width (and, thus, the current density) of the second segment and set $w_1 = 50\,\text{nm}$ and $w_2 = 100\,\text{nm}$, resulting in $j_2 = 0.5 \cdot j_1 = 2\,\text{MA/cm}^2$. Consequently, the equivalent current density reduces to $j_{eq} = (2/3) \cdot j_1 = 2.67\,\text{MA/cm}^2$. Again, the results for the node simulation and the equivalent semi-infinite line match; now we obtain a lifetime of $t_{life} = 8.12\,\text{years}$.

Note that the maximum occurring current density (j_1) remained unchanged – yet the lifetime increased significantly.

Assuming, that the current density j_{Black} is the maximum allowed current density obtained by the fab, we can quantify

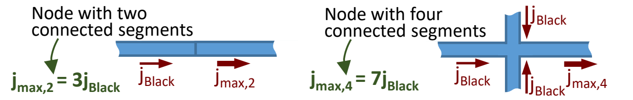

Fig. 6. Increased local current-density limit based on a node's j_{eq}. All segments have the same width and fulfill the stress-length criterion.

the increase of the locally allowed current densities in a multi-segment net. We demonstrate this using the two examples shown in Fig. 6. We can obtain the current density j_{max} by applying Eq. (10) to both cases shown in Fig. 6 where we then set $j_{eq} = j_{Black}$ and solve for j_{max}. This yields $j_{max,2} = 3 \cdot j_{Black}$ for the two-segment node and $j_{max,4} = 7 \cdot j_{Black}$ for the four-segment node, respectively.

V. Summary and Conclusion

In this paper, we proposed an easy-to-apply metric for evaluating the EM robustness of individual nodes within an interconnect. To support this, we derived the stress length criterion, which determines whether the interconnect segments are sufficiently long for our metric to be applicable. For interconnects including segments not fulfilling the length criterion, we showed how EM simulation using RC equivalent circuits can be applied and simplified as well.

Our work aims to pave the way toward physics-based EM verification by enabling manual node lifetime assessment, specifically for segments that exceed the permitted current density given in the PDK. Thus, physics-based modeling can be introduced to practitioners with minimal effort, a capability that is urgently needed to advance EM models and verification.

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