Thermal Modeling of Silicon Photonic Waveguides

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***Abstract*—Photonic devices are essential for high-frequency signal processing, notably in telecommunications. Emerging computational paradigms, such as Ising machines, benefit significantly from the high-speed capabilities of photonic technologies. Coherent Ising machines represent a promising approach to energy-efficient nonclassical computing, specifically by providing rapid solutions to combinatorial optimization problems. The temperature dependence of silicon's optical properties presents a thermal design challenge for large-scale systems. We address this challenge by modeling the thermal behavior of silicon microring resonators to anticipate design constraints for coherent Ising machines. We used a 2D finite element model (FEM) mesh to model the cross-section of the waveguide that constitutes the microring resonator. The simulations were conducted using exclusively open-source software and demonstrate strong agreement with results from other research groups. The thermal relaxation exhibits a bi-exponential behavior, revealing a correlation between the widths of the silicon structure and the thermal relaxation times. Our results indicate that the thermal relaxation times can be varied by thirty percent using geometrical design parameters.**

***Index Terms*—photonic, microring resonator, Ising machine, open source modeling, thermal simulation.**

I. Introduction

Driven by the need for efficient problem-solving methodologies, nonclassical computing has become a significant area of research. In addition to the hyped complex quantum computing methods, photonic annealing processors have emerged as a noteworthy niche technology. Coherent Ising machines (CIMs) offer an energy-efficient alternative for solving a subset of NP-hard problems, including max-cut and graph coloring, which have been successfully mapped and solved using this approach [1]. CIMs do not require extensive cooling to achieve quantum-equivalent states, making them suitable for implementation in smaller-scale devices with significantly lower energy consumption. While significant progress has been made in implementing Ising machines with thousands of spins, a significant gap remains before these systems can be manufactured at scale. To advance Ising machines toward scalable and manufacturable technologies, research is focusing on CMOS-compatible Silicon on Insulator (SoI) technologies, where photonic devices and waveguides are realized in silicon rather than silicon nitride. The principal benefits of this architecture are its compatibility with ongoing microelectronic scaling efforts and the potential for monolithic integration of electronic components. While established time-multiplexing principles utilize a single physical resonator, silicon technology facilitates designs incorporating multiple resonators, with each resonator physically representing an individual spin (Figure 1). A limitation of this technology is the strong thermo-optic effect, or temperature dependence of the refractive index. Several factors must be considered when designing a CIM, with the most critical being thermal behavior, which is essential for stabilizing the spins.

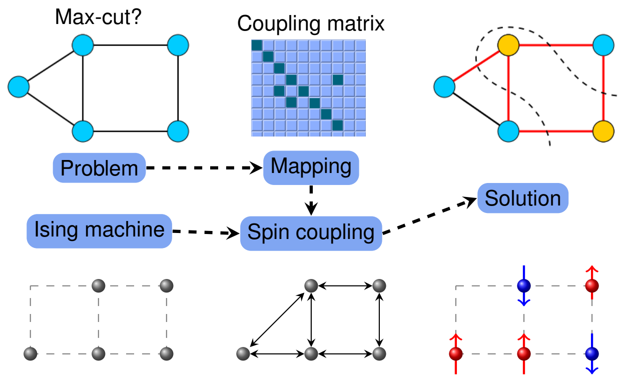

Figure 1. Principle of an Ising machine with multiple spins solving a max-cut problem

The primary contributions of this paper include:

- A simulation study of thermal relaxation times in silicon microring structures,
- Quantification of the impact of design parameters on thermal relaxation time,
- Conclusions on the optimization potential for tuning the thermal relaxation times of microring resonators.

II. State of the Art

Most coherent Ising machines described in literature employ time-multiplexing techniques to achieve a high number of spins for problem-solving [2]. Typically, this involves using a single resonator combined with time-multiplexing with a long optical delay line waveguide that stores the signal pulses of each spin. This approach requires very long waveguides (scaling with the number of simulated spins), thus hindering the development of compact devices. Besides, there are alternative approaches that have been studied focusing on the spatially multiplexed Ising machine built by a network of optical microresonators for solving NP problems. As a consequence, a

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number of microring resonators create Ising spins by utilizing either degenerate optical parametric oscillations or optical bistability, whose conditions can be dominated by the thermo-optic effect in cavities.

Recently, silicon nitride (SiN) has become the predominant material used for waveguide structures and resonators in the field of CIMs. The primary advantage of SiN is its lower thermo-optic coefficient, which results in reduced temperature dependence of the optical properties. Due to the lack of two-photon absorption (TPA), silicon nitride microresonators have less nonlinear loss and heating when operating in nonlinear regimes. However, a significant drawback is that SiN waveguides are only available in specialized photonic semiconductor technologies that are not fully compatible with CMOS processes. In [3], a pair of coupled optical parametric oscillators (OPOs) in SiN have been demonstrated to simulate the ground state of the Ising model.

The use of silicon as a waveguide material facilitates the seamless integration of additional electronic devices, such as transistors and CMOS gates. However, some electronic circuitry is essential for ensuring the function of silicon photonic devices. Typically, a silicon-on-insulator (SoI) technology is employed for silicon photonics applications. The silicon layer atop the buried oxide is patterned to form the waveguide structures for the photonic system. Silicon exhibits a high degree of absorption in its intrinsic state due to the presence of free charge carriers, necessitating electronic depletion to enhance its photonic efficiency. Consequently, doped silicon regions beneath the waveguide are required, which must be electrically connected to create a lateral p-i-n structure with the waveguide situated in the intrinsic region. The electrical control of the p-i-n structure is also used to tune the optical properties of the waveguide during operation.

Silicon-based photonic integrated circuits (ICs) can benefit significantly from the integration of electronics for sensing, computation and electronic control of the photonic device, all fabricated on a single die. This integration enables the development of compact CIM devices, particularly by minimizing the reliance on multiplexing with long delay lines. To date, these devices have primarily been analyzed theoretically [4].

Thermal modeling of silicon waveguides has been conducted by Gray et al. [5] determining the range of thermal relaxation times. Coenen et al. [6] investigate the disparity in thermal relaxation times between heating and cooling phases. While this difference has been previously noted, it has not been adequately explained. Furthermore, this study demonstrates the alteration of time constants resulting from the use of undercuts, where the silicon substrate is partially etched away beneath the photonic structure. These undercuts lead to increased heating and cooling time constants. Therefore, undercuts are not advisable for Ising machine applications, where low relaxation times are desirable.

Indirect methods are required for thermal measurements within photonic devices. Ideally, a resonance frequency measurements can be employed to determine the refractive index of a resonator, thereby identifying the mean temperature of the resonator waveguide. This measurement must be conducted rapidly and without significantly influencing the temperature due to heating effects. Given that thermal relaxation times are of interest, there is a need to explore alternative indirect measurement techniques, such as laser intensity measurements during device operation, which are sensitive to temperature variations.

III. Problem Formulation

Coherent Ising machines operate within relatively narrow ranges of temperature, light frequency, and intensity. For silicon waveguide resonators, a dual-pump OPO is required, utilizing the third-order nonlinearity of silicon to achieve parametric amplification and oscillation. Since pump power is crucial for parametric amplification, it cannot be solely relied upon for temperature tuning. Hence, a separate electrical heating system is required for temperature regulation.

The most critical property of a ring resonator is its resonance frequency. For silicon waveguides this frequency is highly sensitive to temperature variations. Consequently, temperature and its temporal dynamics—specifically, the thermal relaxation time—also become significant properties to consider. In the context of using a resonator as an OPO for an Ising machine, the thermal relaxation time significantly influences the maximum operation speed of the device.

As a CIM operates in pulsed mode to facilitate multiple runs of the ground state estimation for the Ising model, the thermal relaxation time of the waveguide becomes a critical factor influencing the machine's performance. Short thermal relaxation times are desirable, as they enable rapid stabilization of the spins in the ground state and reduce the necessary cooling times between operation cycles, thereby increasing the clock rate of the machine.

Given the complexity of temperature measurement, which often lacks sufficient temporal and spatial resolution, the development of thermal models becomes essential for a comprehensive understanding of temperature variation processes and their influencing factors. This paper will address this issue in the subsequent sections.

IV. Methods

There are multiple approaches to thermal modeling, which can be categorized based on their level of abstraction and mathematical representation. For analyzing the time-dependent thermal behavior of the optical waveguide, we have selected finite element modeling (FEM) and analytical modeling.

A. Finite Element Models

FEM offers the advantage of easily representing complex geometries, as model elements correspond to volume fractions of the actual structure. However, FEM also poses several restrictions and mathematical challenges. To obtain results within a reasonable timeframe, it is necessary to limit the number of nodes and elements in the model.

This can be achieved in several ways: (1) by using a smaller number of nodes per volume, which compromises the accuracy of the results; (2) by reducing the modeled volume; or (3) by simplifying the model's dimensionality, such as opting for 2D instead of 3D representation. In this study, we have employed options (2) and (3). Specifically, we model the silicon waveguide with a constrained volume of surrounding oxide (Figure 2) and conduct simulations in two dimensions. This implies a restriction of the validity to large ring diameters and large distances to the chip boundary. The transient simulation is conducted using 200 equidistant time steps.

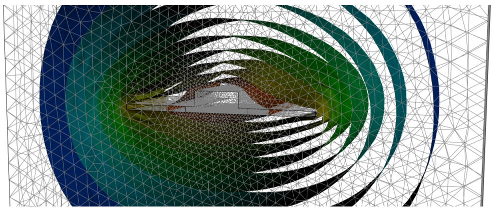

Figure 2. Finite Element model of the waveguide cross-section, showing isothermal surfaces

The simulation of the models were conducted using Elmer FEM [7], [8]. The models themselves were constructed with FreeCAD [9] and gmsh [10] with material parameters from [7], [5] and [11].

B. Analytical Models

The simulation results were analyzed using the Python packages NumPy and SciPy [12], [13]. The findings indicate a bi-exponential (or double exponential) behaviour, as described by Borghi et al. [11], with thermal relaxation times exhibiting minimal dependence on waveguide geometry. In contrast to the findings of Borghi et al., we assert that the bi-exponential behavior is evident in both the heating (see Figure 3, left) and cooling phases (see Figure 3, right). We employ the function

$$T(t) = A \cdot \mathrm{e}^{-\frac{t}{\tau_1}} + B \cdot \mathrm{e}^{-\frac{t}{\tau_2}} + C \qquad (1)$$

to fit the simulated temperature curves, where T and t are temperature and time, respectively; A, B and C are the weighting factors and τ_1 and τ_2 represent the time constants, with $\tau_1 > \tau_2$ indicating the dominant thermal relaxation time. All five parameters $(A, B, C, \tau_1, \tau_2)$ were determined by a nonlinear least squares regression.

V. Experimental Results

As a starting point, we selected a microring design fabricated using the imec iSiPP50G technology [14]. The microring features a diameter of $20\,\mu\mathrm{m}$, with a waveguide cross-section of 220 by $460\,\mathrm{nm}$. We simulated square-wave modulated pump laser pulses with an overall power of $20\,\mathrm{mW}$, assuming that all pump power is converted into heat within the ring resonator. This assumption does not influence the calculated relaxation times significantly, as the thermal system behaves linearly regarding this aspect and the optical time constants are several orders of magnitude lower than the thermal ones. The distinct transient simulations for heating phase and cooling phase each

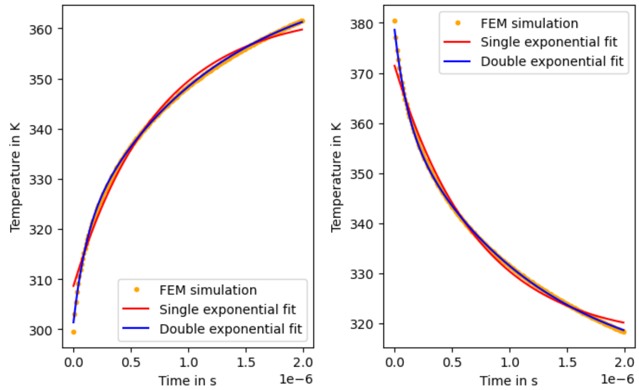

Figure 3. Single and bi-exponential fit for the heating (left) and cooling (right) curve. The single exponential model inadequately describes the thermal behavior.

start from a steady state, while the heating power is switched on or off, respectively, at $t = 0$. The thermal relaxation times are determined using equation (1). Subsequently, we conducted sweeps of the waveguide width and of the silicon width used for electric tuning of the waveguide, as these are the two primary design variables that can be adjusted without altering technology constraints.

Two-dimensional FEM simulations were conducted, showing a dependence of the thermal relaxation time on the geometric design of the waveguide structure (Figure 4).

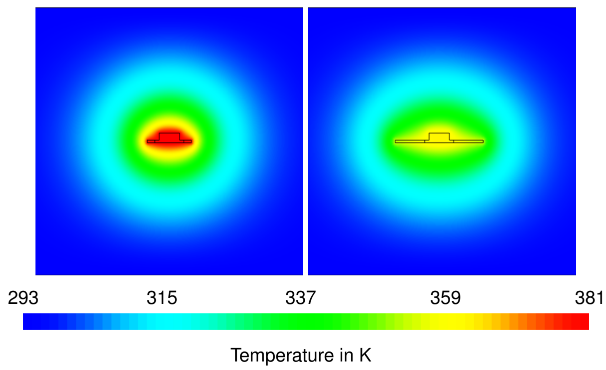

Figure 4. Temperature field around the waveguide cross-section for varying widths of the surrounding thinned silicon, following $2\,\mu\mathrm{s}$ of optical heating of the ring resonator. The wider silicon structure (right, $2\,\mu\mathrm{m}$) facilitates better heat spreading, resulting in a lower waveguide temperature compared to the narrower structure (left, $1\,\mu\mathrm{m}$).

The width of the accompanying SoI stripe significantly influences the thermal relaxation time varying it by over $30\,\%$ (Figure 5). Variations in the waveguide width lead to minimal changes in the thermal relaxation time; specifically, only a $2\,\%$ variation is anticipated for a reasonable range of design parameters (see Figure 6).

The simulation results were verified through comparison with literature [11]. Borghi et al. reported values of $\tau_\mathrm{s} = 1{,}030\,\mathrm{ns}$ for the slow exponential and $\tau_\mathrm{f} = 71\,\mathrm{ns}$ for the fast exponential following multi-objective optimization of their measurement results. Our results indicate a range of $1{,}000$

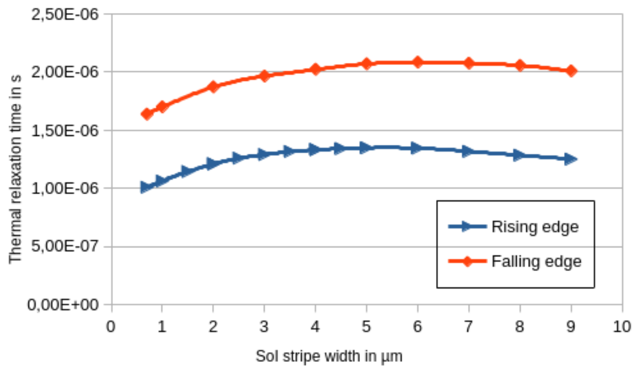

Figure 5. Thermal relaxation time is a function of the SoI stripe width. A narrower stripe width can reduce thermal relaxation times by up to 30 %.

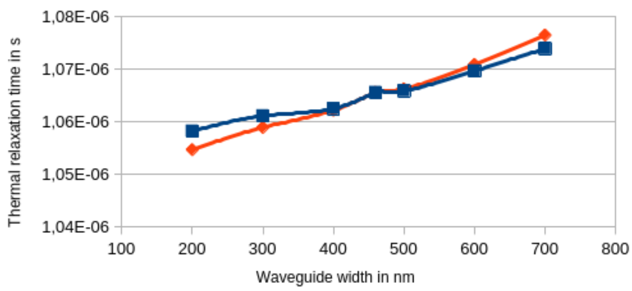

Figure 6. Thermal relaxation time is a function of the waveguide width. The effect is relatively minor, with a realistic variation of only 2 % observed.

to 1,350 ns for τ_s and 48 to 111 ns for τ_f for all considered geometry variations and rising edge. These values align well with those reported in [11].

VI. Conclusion and Outlook

Thermal design poses a significant challenge in the development of optical devices utilizing silicon waveguides. Coherent Ising Machines (CIM) operate in a regime, where the resonance frequency of the oscillators is closely tuned to the desired pump laser frequency. Hence, when constructing a CIM composed with physically distinct oscillators, each oscillator must be identically tuned, regardless of its position and local temperature. The thermal relaxation time is critical for various applications of microring resonators, including CIMs. Thus, a comprehensive understanding of its dependencies is essential. In our study, we investigate the thermal relaxation times and their relation to geometrical design properties that can be directly manipulated by the designer. Notably, variations in layer thickness, which are predetermined by the process design kit (PDK), have been excluded from this analysis.

Our findings indicate that the thermal relaxation time is dependent on the width of the thinner silicon stripe employed for electrical tuning of the waveguide. This suggests that modulation of the thermal relaxation time can be achieved through geometric design alterations, without necessitating changes to the layer structure as defined by the PDK. Furthermore, we demonstrate that the thermal relaxation of a silicon waveguide exhibits a double exponential behavior, with two time constants differing by a factor of 10 to 15.

Recent measurement results revealed excessive noise, hindering effective model validation. To achieve reliable validation, we propose employing a pump-and-probe measurement approach. This method allows for temperature measurements using a weak optical probe signal while simultaneously heating the microring with a powerful pump laser operating at a different light frequency. Additional modeling techniques, such as lumped element and analytical modeling, will be assessed to establish a comprehensive design guideline for integrated photonic devices. Future models will also incorporate metal heater structures positioned above the cladding oxide and the silicon substrate to further investigate their impact on relaxation time.

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